

Proseminar „Das Auswahlaxiom“

WiSe 2015/16

- (1) Mengentheoretische Grundlagen
 - Wichtige Begriffe (Mengen, Relationen, Abbildungen)
 - Axiomatik (BGN vs. ZF[C])
 - Einige Sätze im Zusammenhang mit dem Auswahlaxiom
- (2) Das Konzept der Filter und Ultrafilter
 - Motivation & Definitionen
 - Einige Sätze
 - Mächtigkeit der Familie aller Ultrafilter auf X
 - Zerlegungslemma & Fixpunktsatz
- (3) Die Cauchy'schen Funktionalgleichungen
- (4) [1] Kap. 1.1 - 2.2
- (5) [1] Kap. 3
- (6) [1] Kap. 4.1 -4.3
- (7) [1] Kap. 4.4 (+ evtl. 4.5)
- (8) [1] Kap. 4.6
- (9) [1] Kap. 4.7
- (10) [1] Kap. 4.8
- (11) [1] Kap. 4.9
- (12) [1] Kap. 5.1
- (13) [1] Kap. 5.2
- (14) [1] Kap. 7.1
- (15) [1] Kap. 7.2
- (16) [1] Kap. 6

Literatur

- [1] HERRLICH, H. : *Axiom of choice*. Berlin: Springer, 2006. – xiv + 194 S. <http://dx.doi.org/10.1007/11601562>. – ISBN 3-540-30989-6/pbk

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